Quantitative Analysis

- Moments
- Probability
- Prob. distribution
- Sampling
- Hypothesis Testing
- Correlation & Regression
- Volatility Estimation
- Simulation Modelling
Mean: \( \frac{\sum_{i=1}^{n} X_i}{n} \)

Mode: Value that occurs most frequently

Median: Midpoint of data arranged in ascending/descending order

Variance:

\[ s^2 = \frac{\sum_{i=1}^{n} (X_i - X_{\text{mean}})^2}{n - 1} \]

Population variance

\[ \sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N} \]

Skewness

- Positively: mean > median > mode
- Negatively: mean < median < mode
- Skewness of Normal = 0

Kurtosis

- Leptokurtic: More peaked than normal (fat tails); kurtosis > 3
- Platykurtic: Flatter than a normal; kurtosis < 3
- Kurtosis of Normal = 3
- Excess Kurtosis = Kurtosis - 3

Q. If distributions of returns from financial instruments are leptokurtotic. How does it compare with a normal distribution of the same mean and variance? 

Ans. Leptokurtic refers to a distribution with fatter tails than the normal, which implies greater kurtosis.

\[ \sigma^2 \text{ of return of stock P} = 100.0 \]
\[ \sigma^2 \text{ of return of stock Q} = 225.0 \]
\[ \text{Cov(P,Q)} = 53.2 \]
Current Holding $1 mn in P.
New Holding: shifting $1 million in Q and keeping USD 3 million in stock P. What %age of risk (\( \sigma \)), is reduced?

Ans.

\[ \sigma_f = \sqrt{w^2 \sigma_A^2 + (1-w)^2 \sigma_B^2 + 2w(1-w)\text{Cov(A,B)}} \]

\[ w = 0.75 \]
\[ c^2 = 100^2(0.75)^2 + 225^2(0.25)^2 + 2*0.25*0.75*53.2 \]
\[ \sigma_f = 9.5 \text{ old } \sigma = \sqrt{100} = 10 \]
Reduction = 5%
Quantitative Analysis

Properties

- \( P(A) = \# \text{ of fav. Events} / \# \text{ of Total Events} \)
- \( 0 < P(A) < 1, P(A^c) = 1 - P(A) \)
- \( P(A \cap B) = P(A) + P(B) - P(A \cap B) \)
- \( P(A) + P(B) \) if A, B mutually exclusive
- \( P(A \cap B) = P(A) \cdot P(B) \) if A, B independent
- \( P(A \cap B \cap C) = P(A) \cdot P(B) \cdot P(C) \) if A, B, C mutually independent

Counting principles

- No. of ways to select \( r \) out of \( n \) objects:
  \( ^nC_r = \frac{n!}{r!(n-r)!} \)
- No. of ways to arrange \( r \) objects in \( n \) places:
  \( ^nP_r = \frac{n!}{(n-r)!} \)

Sum rule and Bayes' Theorem

- \( P(B) = P(A \cap B) + P(A^c \cap B) \)
- \( P(B) = P(B \mid A) \cdot P(A) + P(B \mid A^c) \cdot P(A^c) \)
- \( P(B/A) = \frac{P(A \cap B)}{P(A) \cdot P(B) + P(A \cap B)} \)

Q. The subsidiary will default if the parent defaults, but the parent will not necessarily default if the subsidiary defaults. Calculate the probability that both default.

Ans.

\[ P(P \cap S) = P(S/P) \cdot P(P) = 1 \cdot 0.5 = 0.5\% \]

Discrete

Binomial

- Only 2 possible outcomes: failure or success.
- \( P(x) = \binom{n}{x} \cdot p^x \cdot (1-p)^{n-x} \)

Poisson

- Fix the expectation \( \lambda = np \).
- \( P(x) = \frac{\lambda e^{-\lambda}}{x!} \) if \( x = 0 \)
- \( P(x) = 0 \) otherwise

Continuous

Binomial Random Variable

- \( E(X) = np \)
- \( Var(X) = np(1-p) = npq \)

Q. The number of false fire alarms in a suburb of Houston averages 2.1 per day. What is the (approximate) probability that there would be 4 false alarms on 1 day?

Ans.

\[ P(X = x) = \frac{\lambda e^{-\lambda}}{x!} \] if \( x = 0 \)
\[ P(X = 0) = 0 \] otherwise

Q. A portfolio consists of 17 uncorrelated bonds. The 1-year marginal default prob. of each bond is 5.93%. If spread of default prob. is even over the year, calculate the prob. of exactly 2 bonds defaulting in first month?

Ans.

1-month default rate = \( 1 - (1 - 0.593)^{1/12} \)
\[ = 0.00508 \]
Ways to select 2 bonds out of 17 = \( \binom{17}{2} \)
\[ = 17 \cdot 16 / 2 \]
P(Exactly 2 defaults)
\[ = (17 \cdot 16 / 2) \cdot (0.00508)^2 \cdot (1 - 0.00508) \]
\[ = 0.325\% \]
The R.V. X with density function \( f(X) = \frac{1}{b - a} \) for \( a < x < b \), and 0 otherwise, is said to have a uniform distribution over \((a, b)\). Calculate its mean.

**Ans.**

Since the distribution is uniform, the mean is the center of the distribution, which is the average of \( a \) and \( b = \frac{a+b}{2} \)

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At a particular time, the market value of assets of the firm is $100 Mn and the market value of debt is $80 Mn. The standard deviation of assets is $10 Mn. What is the distance to default?

**Ans.**

\[
 z = \frac{A-K}{\sigma_A} = \frac{100-80}{10} = 2
\]
Thank you!

Contact:

E: help@edupristine.com
Ph: +1 347 647 9001