Quantitative Analysis

Time Value of Money
- Future value
  - Value of current cash flow in future – Compounding
    - Amount to which investment grows after one or more time period $FV = PV \times (1 + 1/Y)^N$.
- Present value
  - Present value of future cash flow – Discounting
    - Current value of some future cash flow $PV = FV / (1 + 1/Y)^N$.

Descriptive Statistics

Probability

Probability Distributions

Sampling

Hypothesis Testing

Technical Analysis

Future value

Present value

Annuities
- Series of equal cash flows occurring at evenly spaced interval
- Ordinary annuity: cash flow at end-of-time period
- Annuity due: cash flow at beginning-of-time period
- $PV_{\text{of Annuity Due}} = PV_{\text{of Ordinary Annuity}} \times (1 + r)$
- Perpetuities: annuities with an infinite life
  - $PV_{\text{perpetuity}} = \frac{PMT}{\text{discount rate}}$

Non-Projected Value

$NPV = \sum_{i=0}^{T} \frac{C_i}{(1 + r)^i} - C_0$

NPV is expressed in monetary units ($), IRR is the true interest yield (%age).
In general, NPV is a better measure.

Q: If interest rate of 8%, what will be the value of sum of $1,000 invested today will grow in 5 years?
Ans: $FV = PV \times (1 + 1/Y)^N$
= $1,000 \times (1.08)^5$
= $1,469.3$

Q: If interest rate of 10%, what sum invested today will grow to $1,000 in 5 years?
Ans: $PV = FV \times (1/(1 + r)^N)$
= $(1,000) \times (1/(1.1)^5) = 621$

Q: What is the worth of perpetuity paying $100 annually at an interest rate of 10%?
Ans: $PV_{\text{perpetuity}} = \frac{A}{r}$
= $100/0.1 = $1,000$

We need to know the first three rows of TI BA-II Plus/Professional calculator for CFA Exam.
Discount rate that makes NPV of all cash flows equal to zero

For mutually exclusive projects, NPV and IRR can give conflicting rankings. NPV is a better measure in such cases.

**Q:** If I have to invest today $2,000 for a project which gives me $100 next year, $200 the next, and $250 after that till perpetuity, should I make this investment? Cost of Capital = 10%.

**Ans:**
Value of Perpetuity (At Y2) = 250/0.1 = 2,500

**Q:** A stock is bought today at $10. It pays a dividend of $1 & you sell it at $15 next year. What is the HPR?

**Ans:**
HPR = (15+1-10)/10 = 60%

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IRR

Holding Period Return (Total Return)

Rates of Return on a Portfolio

Money Weighted
• PVs of Cash Inflow = PVs of Cash Outflows
• Solve for discounting rate 'r'

Effective Annual Yield
EAY = (1+HPY)^365/t - 1

Time Weighted
• Form subperiods over the accounting period
• Compute HPR for each subperiod
• Multiply (1+HPR) for each subperiod to get the total return

Bank Discount Yield
R_{BD} = D/F * 360/t

Money Market Yield
r_{MM} = 360 * R_{BD} / (360 – t * R_{BD})

Effective Annual Yield
**Quantitative Analysis**

**Means**

- Arithmetic mean: \( \bar{X} = \frac{\sum_{i=1}^{N} X_i}{N} \)

- Geometric mean: Calculating investment returns over multiple periods or to measure compound growth rates
  \( RG = [(1+R_1) \cdot \ldots \cdot (1+R_n)]^{1/N} - 1 \)

- Harmonic mean: \( \frac{1}{\sum_{i=1}^{N} \frac{1}{X_i}} \)

**Variance & Std. Deviation**

- Average of squared deviations from mean. Population variance:
  \( \sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{N} \)

- Sample variance:
  \( s^2 = \frac{\sum_{i=1}^{N} (X_i - \bar{X})^2}{n-1} \)

- Standard deviation:
  \( \sigma \text{ or } s = \sqrt{\text{Variance}} \)

**Chebyshev’s Inequality:** % of observations lying within k-standard deviations of the mean \( \geq 1 - 1/k^2 \)

**Q:**

ABC was incorporated on Jan 1, 2004. Its expected annual default rate of 10%. Assume a constant quarterly default rate. What is the probability that ABC will not have defaulted by April 1, 2004?

**Ans:**

\[
P(\text{No Default Year}) = P(\text{No def all Quarters})
\]
\[
= (1 - PDQ_1) \cdot (1 - PDQ_2) \cdot (1 - PDQ_3) \cdot (1 - PDQ_4)
\]
\[
PDQ_1 = PDQ_2 = PDQ_3 = PDQ_4 = PDQ
\]

\[
P(\text{No Def Year}) = (1 - PDQ)^4
\]

\[
P(\text{No Def Quarter}) = (0.9)^4 = 97.4\%
\]

**Calculate the standard deviation of following data set:**

- Data Set A: 10, 20, 30, 40, 50
- Data Set B: 10, 20, 70, 120, 130
Expected Return: 
\[ E(X) = P(x_1)x_1 + P(x_2)x_2 + ... + P(x_n)x_n \]

Probabilistic variance: 
\[ \sigma^2(x) = \sum P(x_i) [x_i - E(X)]^2 \]
\[ = P(x_1)[x_1 - E(X)]^2 + P(x_2)[x_2 - E(X)]^2 + ... + P(x_n)[x_n - E(X)]^2 \]

Correlation & Covariance

Correlation = \[ \text{Corr}(R_i, R_j) = \frac{\text{Cov}(R_i, R_j)}{\sigma(R_i) \cdot \sigma(R_j)} \]

Expected return, Variance of 2-stock portfolio:
\[ E(R_p) = w_A E(R_A) + w_B E(R_B) \]
\[ \text{Var}(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B \rho(R_A, R_B) \sigma(R_A) \sigma(R_B) \]

Q:
Amit has invested $300 in Security A, which has a mean return of 15% and standard deviation of 0.4. He has also invested $700 in security B, which has a mean return of 7% and variance of 9%. If the correlation between A and B is 0.4, What is his overall expectation and Standard deviation of portfolio?

Return = 9.4%, Std Deviation = 7.8%
Return = 9.4%, Std Deviation = 24%
Return = 9.4%, Std Deviation = 28%

Ans:
The correct answer is Return = 9.4%, Std Deviation = 24%

\[ \sqrt{w^2 \sigma_A^2 + (1 - w)^2 \sigma_B^2 + 2w(1 - w) \text{Cov}(A, B)} \]

Calculate the correlation between the following data set:
Data Set A: 10,20,30,40,50
Data Set B: 10,20,70,120,130
Sharpe Ratio

Measures excess return per unit of risk. Sharpe ratio = \( \frac{R_p - R_f}{\sigma_p} \)

Roy's safety - First ratio: \( \frac{R_p - R_{target}}{\sigma_p} \)

Sharpe Ratio uses risk free rate, Roys Ratio uses Min. hurdle rate
For both ratios, larger is better.

Coefficient of Variation

Dispersion relative to mean of a distribution; CV=\( \frac{\sigma}{\mu} \)  (\( \sigma \) is std dev.)

Q:
If the threshold return is higher than the risk-free rate, what will be the relationship b/w Roy's safety-first ratio (SF) and Sharpe's ratio?
• Denominator (Sharpe) = Denominator (SF)
• \( R_{target} > R_f \)
• \( R - R_f > R - R_{target} \)
• Sharpe > SF

Ans:
\( R - R_f > R - R_{target} \)

Measurement Scales

Nominal Scale: Observations classified with no order. E.g. Participating Cars assigned numbers from 1 to 10 in the car race.

Ordinal Scale: Observations classified with a particular ranking out of defined set of rankings. E.g. Driver assigned a pole position according to their performance in heats.

Interval Scale: Observations classified with relative ranking. It's an ordinal scale with the constant difference between the scale values. E.g. Average temperature of different circuits.

Ratio Scale: It's an interval scale with a constant ratio of the scale values. True Zero point exists in the ratio scale. E.g. Average speed of the cars during the competition.

Q:
Which of the following type of scale is used when interest rates on Treasury bill is mentioned for 60 years?
A. Ordinal scale
B. Interval scale
C. Ratio scale

Ans: Ratio Scale

Expect 1 questions form Measurement Scales
**Definition & Properties**

- **Empirical probability**: Derived from historical data
- **A Priori probability**: Derived by formal reasoning
- **Subjective probability**: Derived by personal judgment

**Sum Rule and Bayes' Theorem**

- \( P(B) = P(A \cap B) + P(A^c \cap B) = P(B \mid A)*P(A) + P(B \mid A^c)*P(A^c) \)
- \( P(A \mid B) = P(B \mid A)*P(A) / [P(B \mid A)*P(A) + P(B \mid A^c)*P(A^c)] \)

**Questions**

1. The subsidiary will default if the parent defaults, but the parent will not necessarily default if the subsidiary defaults. Calculate the probability of a subsidiary & parent both defaulting. Parent has a PD of 0.5% and subsidiary has PD of 0.9%.
   - **Answer**: \( P(P \cap S) = P(S/P)*P(P) = 1*0.5% = 0.5\% \)

**Dependent and Independent Events**

- A and B are independent if and only if \( P(A \mid B) = P(A) \)
- If the above condition is not satisfied, they are dependent events.
Quantitative Analysis

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Normal Distribution
Binomial Distribution

Normal Distribution
Z-Score
Skewness and Kurtosis

- Continuous Distribution
- Described by mean & variance
- Symmetric about its mean
- Standard Normal Distribution
  - Mean = 0; Variance = 1

Z-score
No. of σ a given observation is away from population mean.
Z=(x-μ)/σ

Quantile:
At a particular time, the market value of assets of the firm is $100 Mn and the market value of debt is $80 Mn. The standard deviation of assets is $10 Mn. What is the distance to default?
Ans:
z = (A-K)/σ_A
   = (100-80)/10
   = 2

Q: Which of the following is likely to be a probability distribution function?
For X=[1,2,3,4,5], Prob[X]= 49/(75-X^2)
For X=[0,5,10,15], Prob[X]=X/30
For X=[1,4,9,16,25], Prob[X]= [(X)^1/2 - 1]/5
Ans:
The correct answer is For X=[0,5,10,15], Prob[X]=X/30
For all values of X, probability lies within [0,1] and sum of all the probabilities is equal to 1.

Q: If Z is a standard normal R.V. An event X is defined to happen if either -1< Z < 1 or Z > 1.5. What is the prob. of event X happening if N(1) = 0.8413, N(0.5) = 0.6915 and N(-1.5) = 0.0668, where N is the CDF of a standard normal variable?
Ans:
P(X)= P(-1< Z < 1) + P(Z > 1.5)
= N(1) - (1-N(1)) + N(-1.5)
= 2*0.8413 - 1 + 0.0668
= 0.7494

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**Quantitative Analysis**

- **Time Value of Money**
- **Descriptive Statistics**
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**Normal Distribution**

- **Skewness**
  - **Positively:** Mean > median > mode
  - **Negatively:** Mean < median < mode

- **Kurtosis**
  - **Leptokurtic:** More peaked than normal (fat tails); excess kurtosis > 0
  - **Platykurtic:** Less peaked / Flatter than a normal; excess kurtosis < 0
  - **Mesokurtic:** Kurtosis of Normal = 3

**Q:** If distributions of returns from financial instruments are leptokurtotic. How does it compare with a normal distribution of the same mean and variance?

**Ans:** Leptokurtic refers to a distribution with fatter tails than the normal, which implies greater kurtosis.
Tracking Error

**Tracking Error**
- Total return on a portfolio (gross of fees) - the total return on the benchmark
- In an Index Fund, the tracking error should be minimal

**Example:** If a portfolio of U.S. stocks has a return of 5% in a period when a comparable U.S. stock index increases by 6% (both on a total return basis), the portfolio's tracking error for that period is -1%

**Roy's Safety First Criterion:**
- For optimal portfolio, minimize SF Ratio,
  - SF Ratio = \[\frac{E(R_p) - R_L}{\sigma_p}\]
- Shortfall Risk = Probability corresponding to SF Ratio

**Uniform Distribution**
- Continuous Distribution
- Outcomes uniformly distributed between \(a\) and \(b\)

**Binomial Distribution**
Discrete Distribution:
• Variables can take 2 values (success / failure)
• Expected Value = \( np \)
• Variance = \( np(1-p) \) (constant)
• Can describe changes in the value of an asset or portfolio
• The probability distribution for a Binomial Random Variable is given by:

\[
P(X=x)=C_n^r p^r(1-p)^{n-r}
\]

• Mean = \( np \), variance = \( np(1-p) \)

Q:
The Prob. of a stock moving up is 0.8 and moving down is 0.2 in a particular day. What is the probability that it would move up at least twice in the 5 working days of the week?

Ans:
\[
P(X \geq 2) = 1-P(X = 0) - P(X = 1)
\]
\[
=1-C_5^0(0.8)^0(0.2)^5 - C_5^1(0.8)^1(0.2)^4
\]
\[
=1-(0.2)^5 - 5*(0.8)^1*(0.2)^4
\]
\[
=0.99328
\]
Quantitative Analysis

Central Limit Theorem

Sampling Distribution

Standard Error (SE)

Probability distribution of all possible sample statistics computed from a set of equal-size samples randomly drawn from the same population.

As Sample Size increases, Sampling Distribution Becomes Almost Normal regardless of shape of population.

Sampling Biases

• Data Mining
• Sample Selection
• Survivorship
• Look-Ahead
• Time-Period

SE (σₜ) of the sample mean is σ of the dist. of sample means
• Known pop. Var. σₓ = σ / √(n)
• Unknown pop. var sₓ = s / √(n)

Q:
25 observation are taken from a sample of unknown variance. Sample mean of 70 and σ = 60. You wish to conduct a 2-tailed test of null hypothesis that the mean is equal to 50. What is most appropriate test statistic?

Ans:
Standard Error of mean (σₓ) = σ / √(n) = 60 / sqrt(25) = 12
Degrees of freedom = 24
Use t-statistic = (x – μ) / σₓ = (70 – 50) / 12 = 1.67

Expect 1 question on the calculation of standard error!!!
Null Hypothesis: $H_0$

Alternative Hypothesis: $H_a$

Confidence Intervals (CI)

Hypothesis Tests for Variances

One Tailed Test

Two Tailed Test

Hypothesis that the researcher wants to reject

Concluded if there is significant evidence to reject $H_0$

Range of values within which $H_0$ Cannot be rejected (say 90% or 95%). Known variance, 2 Tailed test, CI is: $X'' \pm z_{a/2}(\sigma/\sqrt{t})$

Inference Based on Sample Data

Real State of Affairs

$H_0$ is True

$H_0$ is False

Correct decision

Type II error

Confidence level = 1 - $\alpha$

P (Type II error) = $\beta$

Type I error

Significance level = $\alpha^*$

Correct decision

Power = 1 - $\beta$

*Term $\alpha$ represents the maximum probability of committing a Type I error, Type II error cannot be computed easily

Q: Co. ABC would give bonus to employees, if they get a rating higher than 7/10 from customers. A random sample of 30 customers is conducted with rating of 7.1/10. Formulate Hypothesis?

- Null Hypothesis: $H_0$: Mean<=7
- Alternate Hypothesis: $H_a$: Mean>7

Statistic to be measured: t-statistic, with 29 DoF
**Null Hypothesis:**

\( H_0 \)

**Alternative Hypothesis:**

\( H_a \)

**Confidence Intervals (CI):**

Hypothesis Tests for Variances

**One Tailed Test:**

Test if the value is greater than or less than \( K \)

\( H_0: \mu \leq K \) vs. \( H_a: \mu > K \)

**Two Tailed Test:**

Test if the value is different from \( K \)

\( H_0: \mu = 0 \) vs. \( H_a: \mu \neq 0 \)

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**Chi-Square test**

\( \chi^2 = \frac{(n-1)s^2}{\sigma^2} \)

**F test**

\( F = \frac{S_1^2}{S_2^2} \)

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**Q:**

If standard deviation of a normal population is known to be 10 & the mean is hypothesized to be 8. Suppose a sample size of 100 is considered. What is the range of sample means in which hypothesis can be accepted at significance level of 0.05?

**Ans:**

\[ SE = \frac{\sigma}{\sqrt{n}} = \frac{10}{\sqrt{100}} = 1 \]

\[ z = \frac{(x-\mu)}{SE} = \frac{(x-8)}{1} \]

At 95% -1.96<z<1.96

Therefore 6.04<x<9.96
**Quantitative Analysis**

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**Trend Analysis**
- It is based on the observation that market participants tend to act in herds and that trends tend to stay in place for some time.
- In an uptrend, the security's prices go to higher highs and higher lows.
- A downward trend makes lower lows and lower highs.

**Support**:
A low price range in which buying activity is sufficient to stop the decline in price.

**Resistance**:
A high price range in which selling is sufficient to stop the rise in price.

**Change in polarity principle**:
Once a support level is breached, it becomes a resistance level and once a resistance level is breached, it becomes a support level.

- Supply-Demand dictate prices
- Driven by rational & irrational behavior
- Prices move in trends that persist for long periods
- Observe the actual shifts in supply / demand in market prices

**Elliot Wave Theory**
- In a Bull Market
  - An impulse wave consists
    1 = up
    2 = down
    3 = up
    4 = down
    5 = up
  - A Corrective Wave
    a = down
    b = up
    c = down
  - In a Bear Market, the impulse waves are named A-E and the corrective waves are numbered 1-3.

**Fibonacci Sequence**:
- 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ...

**Fibonacci ratios**:
- $\frac{1}{2}=0.5$, $\frac{2}{3}=0.67$, $\frac{3}{5}=0.6$, $\frac{5}{8}=0.625$ etc...
- $2/1=2$, $3/2=1.5$, $5/3=1.67$, $8/5=1.60$, $13/8=1.625$
- The second series of numbers converge to around 1.618, called the Golden Ratio

**Technical Analysis Indicators**

**Price Based Indicators**
- Moving Average Lines – mean of last n closing prices over the last n days
- Bollinger Bands – standard deviation of closing prices over the last n days
- Oscillators
  - Based on market prices, scaled to oscillate around a given value
  - Rate of change oscillators
  - Relative Strength Index
  - Moving Average Convergence/Divergence
  - Stochastic Oscillator

**Sentiment Based Indicators**
- Put/Call Ratio
- Volatility Index
- Margin Debt
- Short Interest Ratio
- Arms Index (TRIN)
- Mutual Fund Cash Position
- New Equity Issuance
Thank you!

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